

# Relating the pion decay constant to the chiral restoration temperature.

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## Abstract

We review the relationship between the pion decay constant  $f_\pi$ , the chiral symmetry restoration temperature  $T_c$  and the phenomenology of low energy chiral symmetry breaking in view of the recent confirmation of the existence of a sigma meson with a mass of 600-700 MeV.

11.30.Qc, 11.30.Rd

## I. INTRODUCTION

Over the past few years many physicists have appreciated the importance of a chiral-symmetry restoring phase transition from nuclear to quark matter. Since the pion decay constant  $f_\pi$  sets the scale for chiral symmetry breaking, it is natural to search for a relationship between  $f_\pi$  and the chiral-symmetry restoring temperature  $T_c$ . The fact that both quantities are of the same order of magnitude indicates that a natural relationship between the two could exist. This relationship was investigated in Refs. [1,2,3] where it was proposed that

$$T_c = 2f_\pi \approx 180\text{MeV} . \quad (1.1)$$

for a pion decay constant  $f_\pi \approx 90$  MeV in the chiral limit. This energy scale (1) is compatible with the estimates from numerical simulations of lattice gauge theories [4,5]. Yet subsequent studies [6,7,8] of  $T_c$  based on the same chiral four-fermion Nambu - Jona-Lasinio (NJL) [9] and linear sigma (LSM) [10] models used in part to derive (1) find instead  $T_c = \sqrt{2}f_\pi$  or  $f_\pi$ .

In this paper we return to these two chiral models and demonstrate that the preferred  $T_c$  in (1) is directly correlated to the NJL zero temperature and chiral-limiting (CL) scalar sigma mass [9]

$$m_\sigma = 2m_q . \quad (1.2)$$

Here  $m_q$  is the CL nonstrange (constituent) quark mass near  $m_q \approx M_N/3 \sim 300$  MeV. Since a very recent measurement of  $m_\sigma$  finds [11] a clear enhancement in the invariant  $\pi^0\pi^0$  mass around 650 MeV, supporting earlier observations [12], this leaves little doubt that the  $\sigma$  (600-700) really does exist and that the NJL-LSM  $\sigma$  meson in (2) is **not** just a “toy” particle. It is therefore an appropriate time to return to a study of (1) based on the NJL relation (2).

In Section 2 we review the NJL scheme, primarily focusing on the relation between  $T_c$  and the ultra-violet cutoff  $\Lambda$ . Keeping careful track of the expansion terms in quark mass

relative to the cutoff  $\Lambda$ , one obtains Eq. (1) after some straightforward algebra. In Section 3 we work with a linearized chiral quark model (CQM) lagrangian to obtain the same results as in the NJL framework. In fact the latter CQM approach can be used to dynamically generate the complete LSM lagrangian (including quartic terms) of Gell-Mann and Lévy. Finally in Section 4 we briefly recall the BCS analogy [1] between energy gap and critical temperature. Such an approach is extended to QCD. In all cases in Sections 2-4 the relation  $T_c \simeq 2f_\pi$  is obtained as noted in the concluding Section 5.

## II. FOUR FERMION NJL MODEL

We will start our discussion about the relationship between  $T_c$  and  $f_\pi$  in the NJL model. We will show that the correct relationship is indeed Eq. (1). To make the point clearly we adhere in this section to the notation used in Ref. [7]. The lagrangian density is given by

$$\mathcal{L}_{NJL} = \bar{\psi} i \not{\partial} \psi + G \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right] , \quad (2.1)$$

where the spinor  $\psi$  denotes an isodoublet of u and d quark fields. The standard NJL gap equation for the nonstrange quark mass depicted in Fig. 1 is given by

$$m_q = \tilde{G} \int \frac{d^4 p}{(2\pi)^4} \text{Tr}[S_F(p)] , \quad (2.2)$$

where in the notation of Ref. [7]  $\tilde{G} \equiv G(2N_c N_f + 1)$ .

This is generalized to finite temperatures, using the standard procedure, i.e.,  $S_F(p)$  is replaced by the temperature-dependent propagator

$$S_F(p, T) = \frac{i(\not{p} + m)}{p^2 - m^2} - \frac{2\pi\delta(p^2 - m^2)(\not{p} + m)}{e^{|p_0/T|} + 1} , \quad (2.3)$$

and the quark mass  $m_q$  in (4) is replaced by a temperature- dependent mass  $m_q(T)$ . As was shown by Dolan and Jackiw [13], this is equivalent to using the discrete sum over energies in the Matsubara formalism [14]. After these two replacements for  $m_q$  and  $S_F$  in (4) are made and the integral over  $p_0$  is performed, the temperature-dependent gap equation becomes

$$m_q(T) = 4\tilde{G} \int \frac{d^3p}{(2\pi)^3} \frac{m_q(T)}{2E_p} \left[ 1 - \frac{2}{e^{E_p/T} + 1} \right] , \quad (2.4)$$

where  $E_p \equiv \sqrt{p^2 + m_q^2(T)}$ . A common factor of  $m_q(T)$  can be cancelled on both sides of Eq. (6). In the limit where one reaches the transition point for chiral-symmetry restoration, the quark mass “melts” and Eq. (6) becomes

$$1 = 2\tilde{G} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} \left[ 1 - \frac{2}{e^{p/T_c} + 1} \right] . \quad (2.5)$$

The integral in (7) can be easily evaluated using a three-dimensional (3D) cutoff  $\Lambda$ , leading to

$$1 = 2\tilde{G} \left[ \frac{\Lambda^2}{4\pi^2} - \frac{T_c^2}{12} \right] , \quad (2.6)$$

or

$$\frac{T_c^2 \pi^2}{3} = \Lambda^2 - \frac{2\pi^2}{\tilde{G}} . \quad (2.7)$$

We observe that a linear relation is established in (9) between the cutoff  $\Lambda^2$  and the critical temperature  $T_c^2$ . For further reference we note that the integration of the gap equation (4) at zero temperature gives in terms of this 3D cutoff  $\Lambda$

$$1 = \frac{2\tilde{G}}{4\pi^2} \left[ \Lambda \sqrt{\Lambda^2 + m_q^2} - m_q^2 \ln \left( \frac{\Lambda}{m_q} + \sqrt{1 + \frac{\Lambda^2}{m_q^2}} \right) \right] . \quad (2.8)$$

To find the relation between the critical temperature  $T_c$ , and the pion decay constant  $f_\pi$ , we now turn our attention to the (logarithmically divergent) “gap equation” derived from the quark loop for  $\langle 0 | A_\mu | \pi \rangle = i f_\pi q_\mu$  in the chiral limit as

$$f_\pi^2 = -4i N_c m_q^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m_q^2)^2} . \quad (2.9)$$

Upon integration, (11) leads to

$$f_\pi^2 = \frac{N_c m_q^2}{2\pi^2} \left[ \ln \left( \frac{\Lambda}{m_q} + \sqrt{1 + \frac{\Lambda^2}{m_q^2}} \right) - \frac{\Lambda}{\sqrt{\Lambda^2 + m_q^2}} \right] . \quad (2.10)$$

Combining (12) with the zero-temperature gap equation (10) one obtains

$$\frac{4\pi^2}{2\tilde{G}} + \frac{2\pi^2 f_\pi^2}{N_c} = \frac{\Lambda^3}{\sqrt{\Lambda^2 + m_q^2}} . \quad (2.11)$$

To proceed further it is necessary to write the right hand side (rhs) of (13) in a Taylor series expansion as

$$\frac{\Lambda^3}{\sqrt{\Lambda^2 + m_q^2}} \approx \Lambda^2 - \frac{1}{2}m_q^2 + \frac{3}{8}\frac{m_q^4}{\Lambda^2} + \dots \quad (2.12)$$

Now substituting the cutoff  $\Lambda^2$  in (13) into the expression for the critical temperature in (9) one finds

$$\frac{4\pi^2}{2\tilde{G}} + \frac{2\pi^2 f_\pi^2}{N_c} = \frac{T_c^2 \pi^2}{3} + \frac{2\pi^2}{\tilde{G}} - \frac{1}{2}m_q^2 - \frac{3}{8}\frac{m_q^4}{\Lambda^2} + \dots , \quad (2.13)$$

with terms of  $m_q^6/\Lambda^4$  being neglected . The last equation can be written in an almost cutoff-free form

$$T_c^2 = 2f_\pi^2 \frac{3}{N_c} + \frac{3}{2\pi^2}m_q^2 - \frac{9}{8\pi^2}\frac{m_q^4}{\Lambda^2} + \dots , \quad (2.14)$$

a result which was first obtained by Kocić [15] (to leading order).

Upon using the quark-level Goldberger-Treiman (GT) relation

$$m_q = f_\pi g , \quad (2.15a)$$

along with the dimensionless pion-quark coupling constant [16]

$$g = \frac{2\pi}{\sqrt{N_c}} , \quad (2.15b)$$

Eq. (16) becomes (for  $N_c = 3$ )

$$T_c^2 = 2f_\pi^2 + 2f_\pi^2 - \frac{9}{8\pi^2}\frac{m_q^4}{\Lambda^2} + \dots . \quad (2.16)$$

Finally dropping the small  $\mathcal{O}(m_q^4/\Lambda^2)$  term, Eq. (18) gives the critical temperature

$$T_c \simeq 2f_\pi \approx 180\text{MeV} . \quad (2.17)$$

Keeping the last term in Eq. (18) reduces  $T_c$  from 180 MeV to 172 MeV. Although (19) was originally determined in an NJL scheme in Ref. [2], it also follows in the recent review [7] if all leading order terms are consistently kept.

### III. NJL - LSM MODEL

The above NJL analysis scaled to a 3-dimensional cutoff can in fact be formulated in the equivalent language of linear  $\sigma$  and  $\pi$  auxiliary (pseudo-elementary) fields with the following lagrangian density exploited by Eguchi [17]

$$\mathcal{L}_E = \bar{\psi} i \not{\partial} \psi + g \bar{\psi} (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi - \frac{\mu_\sigma^2}{2} [\sigma^2 + \pi^2] . \quad (3.1)$$

Using path integral methods it has been shown that the lagrangians (3) and (20) are identical provided the dimensional coupling  $G$  of the NJL model is related to the dimensionless meson-quark coupling  $g$  as  $G = g^2/2\mu_\sigma^2$ . With hindsight, the crucial relation used here (17b), may appear to be outside the framework of the NJL model. But its relation to the NJL coupling  $G$  suggests

$$\tilde{G} \approx N_c/2f_\pi^2 . \quad (3.2)$$

This means that  $\tilde{G}$  scaling the mass gap equation (4), is itself scaled to (21). Furthermore since  $f_\pi \sim N_c^{1/2}$ ,  $\tilde{G}$  is independent of  $N_c$ , as expected.

Recently the equivalence between (3) and (20) has been used by Delbourgo and Scadron [18] to combine the features of the NJL model with some of those of the linear  $\sigma$  model. In this chiral quark model (CQM) context the lagrangian density has the chiral-invariant form closely related to (20)

$$\mathcal{L}_{CQM} = \bar{\psi} i \not{\partial} \psi + \frac{1}{2} [(\partial\sigma)^2 + (\partial\pi)^2] + g \bar{\psi} (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi . \quad (3.3)$$

In addition one requires the validity of a mass gap equation simulating the NJL Eq. (4) but also including Eq. (11). So we consider quark mass generation in the dynamically generated CQM-LSM framework. Following Refs. [18,19], the log-divergent “gap equation” derived from the quark loop for  $\langle 0|A_\mu|\pi\rangle = i f_\pi q_\mu$  for decay constant  $f_\pi = m_q/g$  in the CL is (defining  $\bar{d}^4 p = (2\pi)^{-4} d^4 p$ )

$$1 = -i 4 N_c g^2 \int \bar{d}^4 p (p^2 - m_q^2)^{-2} . \quad (3.4a)$$

The above nonperturbative equation for  $\delta f_\pi = f_\pi$  should be correlated with  $\delta m_q = m_q$  obtained from the quadratically divergent [18] quark “tadpole” graph of Fig. 2 for  $u$  and  $d$  quark flavours and colour number  $N_c = 3$ , leading to the “mass gap”

$$m_q = \frac{i \, 8 \, N_c \, g^2}{-m_\sigma^2} \int \frac{\bar{d}^4 p \, m_q}{p^2 - m_q^2}. \quad (3.4b)$$

The NJL mass gap (4) and the CQM mass gap (23b) are consistent with the NJL coupling (21).

Note that  $m_q$  in (23b) is a counter-term mass in the kinetic part of the chiral symmetric CQM lagrangian  $i\bar{\psi} \rightarrow i\bar{\psi} - m_q + “m_q”$ . In this self-consistent approach the first  $-m_q$  term signals the quark loop in Fig. 2 to have mass  $m_q$  while the second  $+ “m_q”$  counter-term mass on the lhs of (23b) is computed from the quark loop, itself on the rhs of (23b). Finally, one uses the dimensional regularization identity in  $2\ell = 4$  dimensions [18]

$$\int \frac{\bar{d}^4 p}{p^2 - m^2} - \int \frac{\bar{d}^4 p \, m^2}{(p^2 - m^2)^2} = \lim_{\ell \rightarrow 2} \frac{-i \, m^{2\ell-2}}{(4\pi)^\ell} [\Gamma(1 - \ell) + \Gamma(2 - \ell)] = \frac{i \, m^2}{(4\pi)^2}, \quad (3.5)$$

together with the log-divergent gap equation (23a). Then the mass gap (23b) becomes replaced by

$$1 = \frac{2 \, m_q^2}{m_\sigma^2} \left[ 1 + \frac{g^2 \, N_c}{4\pi^2} \right]. \quad (3.6)$$

Mass counter-terms coupled with the dimensional regularization identity (24) lead to cancelling signs which then scales mass in the CQM in a manner analogous to mass generation in the NJL scheme. Only now 4D cutoffs can be explicitly avoided, whereas in the NJL analysis in Section 2, the use of 3D cutoffs enters the theory.

In a similar manner to the quark loop in Fig. 2, the scalar (counter-term) mass  $m_\sigma^2$  is self-consistently determined [18] by the  $\sigma$  quark “bubble” and tadpole graphs of Fig. 3 leading to the CL value

$$m_\sigma^2 = 8iN_c \, g^2 \int \frac{\bar{d}^4 p \, (p^2 + m_q^2)}{(p^2 - m_q^2)^2} - 3! \frac{8 \, N_c \, g \, g' \, m_q}{m_\sigma^2} \int \frac{\bar{d}^4 p}{p^2 - m_q^2} \quad (3.7a)$$

$$= 16 \, i \, N_c \, g^2 \left[ \int \frac{\bar{d}^4 p \, m_q^2}{(p^2 - m_q^2)^2} - \int \frac{\bar{d}^4 p}{p^2 - m_q^2} \right] \quad (3.7b)$$

$$= \frac{N_c \, g^2 \, m_q^2}{\pi^2}. \quad (3.7c)$$

To proceed from (26a) to (26b) we have applied the meson-meson coupling  $g' = m_\sigma^2/2f_\pi$  needed to keep  $m_\pi = 0$  in the LSM [10]. Then (26c) follows from (26b) using the dimensional regularization identity (24). Finally the two equations (25) and (26c) can be solved for the two unknowns (for  $N_c = 3$ ) as [18]

$$m_\sigma = 2m_q \quad (3.8a)$$

$$g = 2\pi/\sqrt{3} \approx 3.6276. \quad (3.8b)$$

Since the quark tadpole graph of Fig. 2 is the obvious extension of the NJL four-quark Hartree loop of Fig. 1, it should not be surprising that both the dynamically generated NJL and LSM-type theories lead to (27a).

From the perspective of the CQM lagrangian (22), the meson-quark coupling  $g$  is also self-consistently determined in (27b). In fact this coupling  $g \approx 3.63$  in (27b) makes direct contact with the empirically deduced [20]  $\pi NN$  pseudoscalar coupling  $g_{\pi NN} \approx 13.08$ , since the latter implies

$$g = g_{\pi NN}/3g_A \approx 3.55 \quad (3.9)$$

for 3 quarks in a nucleon, with [21]  $g_A \approx 1.2573$  a three-body effect. But from our point of view, the most important consequence of (27b) is that this CQM coupling constant  $g$  correctly sets the scale of the CL quark and sigma masses via  $f_\pi \approx 90$  MeV as [18]

$$m_q = (2\pi/\sqrt{3})f_\pi \approx 325 \text{ MeV} \quad (3.10a)$$

$$m_\sigma = 2m_q = (4\pi/\sqrt{3})f_\pi \approx 650 \text{ MeV}. \quad (3.10b)$$

Not only does (29b) comply with recent data [11] on  $m_\sigma$ , but (29a) is close to the usual quark model value  $M_N/3$  and also near the gauge-parameter independent nonperturbative QCD dynamical value of [16]  $m_{dyn} \approx [4\pi\alpha_s\langle-\bar{q}q\rangle/3]^{1/3} \approx 320$  MeV for the usual quark condensate  $\langle-\bar{q}q\rangle^{1/3} \approx 250$  MeV at momentum scale 1 GeV where [22]  $\alpha_s(1 \text{ GeV}^2) \approx 0.5$ .



As for the original chiral-invariant linear sigma model (LSM) of Gell-Mann and Lévy [10], starting from the CQM lagrangian (22) the shifted interacting part of the LSM lagrangian can be self-consistently induced at tree level as

$$\mathcal{L}_{int} = g\bar{\psi}(\sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi})\psi + g'\sigma(\sigma^2 + \vec{\pi}^2) - (\lambda/4)(\sigma^2 + \vec{\pi}^2)^2, \quad (3.11a)$$

with chiral couplings

$$g = m_q/f_\pi, \quad g' = \lambda f_\pi = m_\sigma^2/2f_\pi. \quad (3.11b)$$

The first meson-quark coupling term in (30) is self-consistently determined from the CQM lagrangian (22). However the mesonic cubic and quartic LSM couplings in (30) obtained in tree order are again induced in one-loop order via the log-divergent gap equation (23a) as [18]

$$\begin{aligned} g_{\sigma\pi\pi} &= -i \, 2 \, N_c \, g^3 \int \frac{\bar{d}^4 p \, \text{Tr}(\not{p} + m_q)(p^2 - m_q^2)}{(p^2 - m_q^2)^3} \\ &= 2m_q \, g \left[ -i \, 4 \, N_c \, g^2 \int \bar{d}^4 p \, [p^2 - m_q^2]^{-2} \right] \\ &= 2m_q g = g', \end{aligned} \quad (3.12a)$$

$$\lambda = 2 \, g^2 \left[ -i \, 4 \, N_c \, g^2 \int \bar{d}^4 p \, [p^2 - m_q^2]^{-2} \right] = 2g^2, \quad (3.12b)$$

provided  $f_\pi g = m_q$  and  $m_\sigma = 2m_q$ . In effect equations (31) correspond to the “compositeness condition” [23]  $Z = 0$ , treating the  $\pi$  and  $\sigma$  states as bound (in the NJL model) or as elementary (in the CQM or LSM theories).

The authors of Ref. [8] considered a similar starting point, based on the linear  $\sigma$  model. The gap equation (23a) is obtained via the condition that  $Z_3 = 0$  (also see Ref. [24]), implicitly leading to (31a). However imposing an additional compositeness condition  $Z_4 = 0$ , Ref. [8] do not obtain (31b). Instead they find  $\lambda = g^2$ ,  $T_c \simeq f_\pi$  along with  $m_\sigma^2 = 2m_q^2 + m_\pi^2$ . All of these results are in disagreement with our conclusions above. In fact their latter result is not consistent with the standard form  $m_\sigma^2 = 4m_q^2 + m_\pi^2$  and this suggests as noted by the

authors of Ref. [8] that a complete LSM has a  $\lambda\sigma^4$  term with meson loops which may not be compatible with a  $Z_4 = 0$  compositeness condition.

Another approach to the strict LSM based on the Lee null tadpole condition [25] was considered in Ref. [6]. Although this analysis also finds  $T_c \approx f_\pi$ , the authors of Ref. [6] also note that this result is “physically not relevant in the high-temperature region”.

As for finite temperature effects in the CQM-LSM framework, at zero chemical potential the chiral symmetry restoration temperature  $T_c$  should be the same whether the quark mass  $m_q$  or the sigma mass  $m_\sigma$  “melts” to zero. We limit our considerations to the melting of the quark tadpole mass of Fig. 2, resulting in

$$m_q \rightarrow m_q(T_c) = m_q + \frac{8 N_c g^2 m_q}{-m_\sigma^2} \left( \frac{T_c^2}{2\pi^2} \right) J_+(0) , \quad (3.13)$$

where  $J_+(0)$  is one of the exponential integrals

$$J_+(0) = \int_0^\infty x dx (e^x + 1)^{-1} = \pi^2/12 , \quad (3.14)$$

so that  $(T_c^2/2\pi^2) J_+(0) = T_c^2/24$  for fermion loops. The minus sign in the denominator of (32) is due to the  $(q^2 - m_\sigma^2)^{-1}$  propagator structure of the tadpole at  $q^2 = 0$  and the two minus signs from the fermion loop and from the antiperiodicity condition for fermions cancel for fermion loops. Since the quark mass melts at  $m_q(T_c)=0$ , and the  $T = 0$   $m_q$  factor divides out of (32), one then obtains

$$m_\sigma^2 = g^2 T_c^2 \quad \text{or} \quad T_c = 2f_\pi , \quad (3.15)$$

for  $N_f = 2$ ,  $N_c = 3$ , along with  $m_\sigma = 2m_q = 2f_\pi g$ .

Alternatively in a spontaneously generated LSM framework, a  $\sigma$  mass not constrained to  $2m_q$  melts to zero to give once again [1]  $T_c = 2f_\pi$ . This also follows from the LSM analysis [2] for  $m_\sigma^2(T_c) = 0$  if the quark tadpole of Fig. (2) is kept. We believe it significant that whether melting the quark mass as in (32) or melting the  $\sigma$  mass as in Refs. [1] or [2],  $T_c$  is always  $2f_\pi \approx 180$  MeV for the CL pion decay constant  $f_\pi \approx 90$  MeV.

#### IV. BCS APPROACH

Here we offer one final justification of  $T_c = 2f_\pi$  based on the original BCS approach to low-temperature superconductivity [26]. As noted in Refs. [1,2], BCS cut off a non-relativistic gap integral

$$1 = \lambda_D \int^{\omega_D} \frac{d^3p}{(2\pi)^3 2E} \tanh \frac{E}{2T} \quad (4.1)$$

at the Debye energy  $\omega_D \gg T_c$ . Then BCS made an asymptotic expansion in the the energy gap ( $\Delta_0$ ) relative to  $T_c$  to obtain the dimensionless ratio

$$2\Delta_0/T_c = 2\pi e^{-\gamma_E} \approx 3.52 . \quad (4.2a)$$

But this BCS ratio (36a) is strikingly similar to the relativistic Goldberger-Treiman ratio at the quark level for the NJL-LSM

$$m_q/f_\pi = g = 2\pi/\sqrt{3} \approx 3.63 , \quad (4.2b)$$

provided one replaces  $\Delta_0$  in (36a) by  $m_q$  in (36b) and **identifies**  $T_c$  **with**  $2f_\pi$  (or  $T_c \approx 180$  MeV).

Also, an analogue (but unmeasured) Debye high energy cutoff  $\omega_D$  for a constant  $m_q$  numerically requires [1]  $T_c \approx 176$  MeV. However, a “running mass” QCD modification eliminates the need for this  $\omega_D$  cutoff but still results [1] in a (melting) chiral temperature  $T_c \approx 170$  MeV via a gap-type equation similar to (35) (using  $E_k(T) = [\underline{k}^2 + m_{dyn}^2(T)]^{\frac{1}{2}}$ )

$$1 = \frac{2\alpha_s}{\pi} \left[ \int_0^{m_{dyn}} \frac{dk}{E_k(T_c)} \tanh \frac{E_k(T_c)}{2T_c} + \int_{m_{dyn}}^\infty \frac{dk}{E_k(T_c)} \frac{m_{dyn}^2}{\underline{k}^2} \tanh \frac{E_k(T_c)}{2T_c} \right. \\ \left. + \int_{m_{dyn}}^\infty \frac{dk}{E_k(T_c)} \left( \frac{m_{dyn}}{2|\underline{k}| - m_{dyn}} - \frac{m_{dyn}^2}{\underline{k}^2} \right) \right] , \quad (4.3)$$

for the dynamically generated QCD quark mass [26]  $m_{dyn} \approx 320$  MeV.

The important point to note is that a nonrelativistic BCS asymptotic expansion (36a) involves Cooper-paired acoustical phonons with  $E \sim p$ . Likewise a tightly bound  $\bar{q}q$  pion couples to quarks in (36b) via relativistic gluon exchange with  $E \sim p$ . Consequently it should not be surprising that the energy gap to critical temperature ratios in (36) strongly suggest  $T_c = 2f_\pi$  in the relativistic case (36b).

## V. CONCLUSIONS

In conclusion, we have reviewed different proposals made recently in the literature about the relationship between two basic quantities appearing in theories of chiral symmetry, namely the pion decay constant  $f_\pi$  and the chiral symmetry restoration temperature  $T_c$ . The result which best describes the analysis of lattice gauge theories is  $T_c = 2f_\pi$  and we have discussed various ways in Sections 2-4 to justify this relationship. The recent confirmation of the  $\sigma$ -meson of a mass approximately 650 MeV is a very important road indicator towards the correct description of low energy phenomena using models based on chiral symmetry.

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## FIGURES

FIG. 1. Quark mass generation in Hartree approximation in the NJL model.

FIG. 2. Quark tadpole diagram representing quark mass  $m_q$  in the CQM-LSM.

FIG. 3. Quark bubble (a) and tadpole diagram (b) representing squared  $\sigma$  mass  $m_\sigma^2$ .





Figure 1



Figure 2

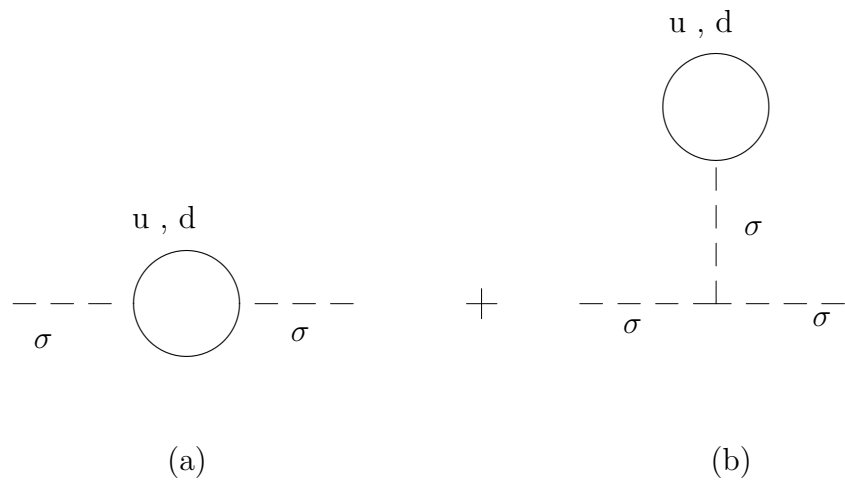


Figure 3

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402201v2>

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